Complex   سكشن رياضة ٣

* Roots of complex number :-

$$Z_K = (x+iy)^{\frac{1}{n}} = r^{\frac{1}{n}}\, e^{i\left(\frac{\theta+2\pi K}{n}\right)}$$

* Find solution / Find value / Find roots

$\boxed{1}$ $$Z_K = (-2-2\sqrt{3}\,i)^{\frac{1}{4}} = r^{\frac{1}{4}}\, e^{i\left(\frac{\theta+2\pi K}{n}\right)}$$

$K = 0, 1, 2, 3$

$$r = |Z| = \sqrt{x^2+y^2} = \sqrt{4+12} = 4$$

$$\theta = \pi + \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \frac{4\pi}{3}$$

So $$Z_K = (4)^{\frac{1}{4}} \cdot e^{i\left(\frac{\frac{4\pi}{3}+2K\pi}{n}\right)}$$

$$Z_K = \sqrt{2}\left[\cos\left(\frac{\frac{4\pi}{3}+2K\pi}{4}\right) + i\sin\left(\frac{\frac{4\pi}{3}+2K\pi}{4}\right)\right.$$

$K=0 \Rightarrow Z_0 = \frac{1}{\sqrt{2}} + \frac{\sqrt{6}}{2} i$

$K=1 \Rightarrow Z_1 =$

$K_2 \Rightarrow Z_2 = \frac{-1}{\sqrt{2}} - \frac{\sqrt{6}}{2} i$

$K_3 \Rightarrow Z_3 = \frac{\sqrt{6}}{2} - \frac{1}{\sqrt{2}} i$

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*Find solution $Z^{\frac{3}{2}} = 4\sqrt{2} + i4\sqrt{2}$

$\therefore Z_K = (4\sqrt{2} + i\,4\sqrt{2})^{\frac{3}{2}}$

$= r^{\frac{2}{3}} \cdot e^{i\left(\frac{\theta + 4\pi K}{3}\right)}$

$r = |z| = \sqrt{x^2 + y^2} = \boxed{8}$

$\theta = \tan^{-1} \frac{4\sqrt{2}}{4\sqrt{2}} = \frac{\pi}{4}$

$\therefore Z_K = (8)^{\frac{2}{3}} \cdot e^{i\left(\frac{\frac{\pi}{4} + 4K\pi}{3}\right)}$

$$Z_K = (8)^{\frac{2}{3}}\left[\cos\left(\frac{\frac{\pi}{4}+4K\pi}{3}\right)+i\sin\left(\frac{\frac{\pi}{4}+4K\pi}{3}\right)\right]$$

$K=0 \Rightarrow Z_0 = 2\sqrt{3}+2i$

$K=1 \Rightarrow Z_1 = -4i$

$K=2 \Rightarrow Z_2 = -2\sqrt{3}+2i$

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* $Z^3 - 1 = 0 \longrightarrow Z^3 = 1 \longrightarrow Z = (1)^{\frac{1}{3}}$

$$Z_K = (1+0i)^{\frac{1}{3}} = r^{\frac{1}{3}} \cdot e^{i\left(\frac{\theta+2K\pi}{3}\right)}$$

$r = |Z| = \sqrt{x^2+y^2} = \sqrt{1+0} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$$Z_K = 1 \cdot e^{i\left(\frac{2K\pi}{3}\right)} = \cos\left(\frac{2K\pi}{3}\right)+i\sin\left(\frac{2K\pi}{3}\right)$$

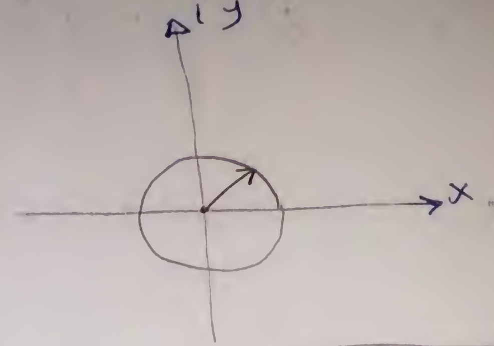
$K=0 \longrightarrow Z_0 = 1+i0$

$K=1 \longrightarrow Z_1 = \frac{\sqrt{3}}{2}+i\frac{1}{2}$

$K=2 \longrightarrow Z_2 = \frac{\sqrt{3}}{2}-i\frac{1}{2}$

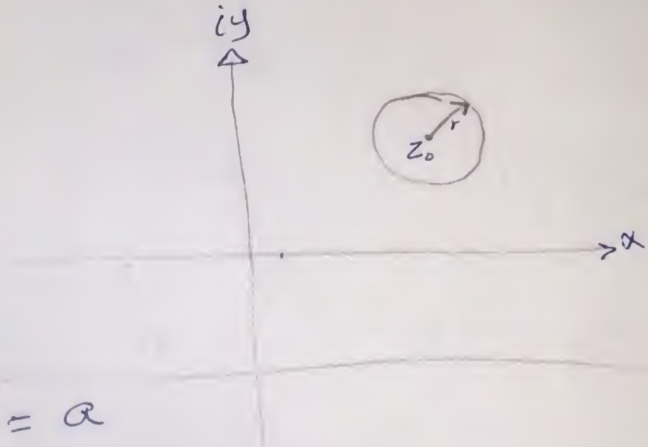
* Some Curves in Complex Plane :-

$\boxed{1}$ $|Z| = r$



← دائرة نصف قطرها r ومركزها نقطة الأصل.

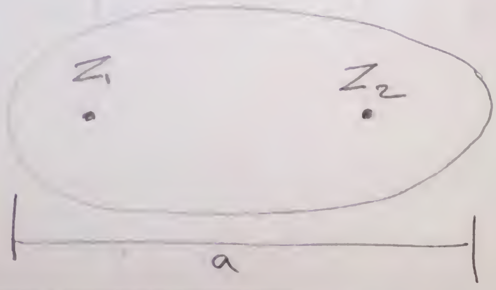
$\boxed{2}$ $|Z - Z_0| = r$



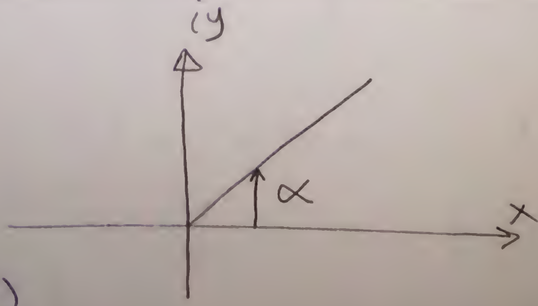
← دائرة نصف قطرها r ومركزها Z_0.

$\boxed{3}$ $|Z - Z_1| + |Z_1 - Z_2| = a$

قطع ناقص



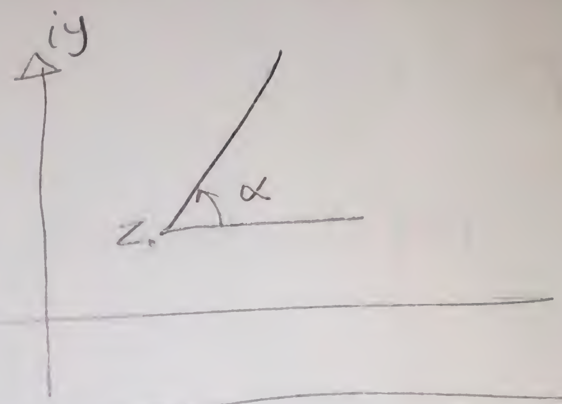
$\boxed{4}$ $\arg(Z) = \alpha$



← هي جميع النقاط الواقعة على الخط المستقيم الذي يصنع زاوية (α) مع محور السينات عند نقطة الأصل

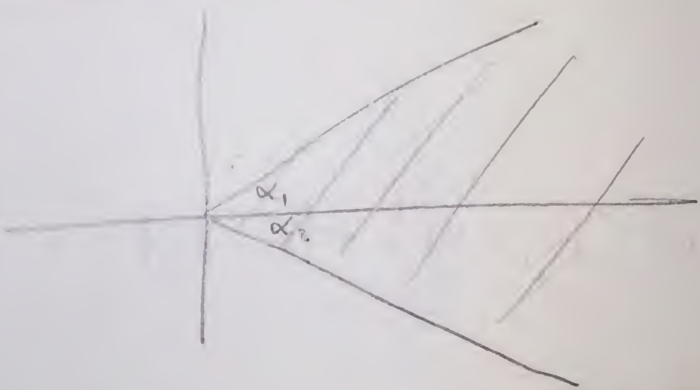
$\boxed{5}$ $\arg(Z - Z_0) = \alpha$

← نفس الكلام ولكن من نقطة Z .



$\boxed{6}$ $|\arg(z)| \leq \alpha$

$-\alpha \leq \arg(z) \leq \alpha$



هام

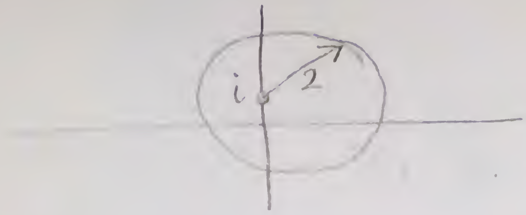
① لو $<$ أو $>$ نرسم المنحنى فقط .

② المعادلة ليست كالسابقة نعوض عن $(Z = x + iy)$ ونشوف معادلة المنحنى الناتج تمثل إيه ؟

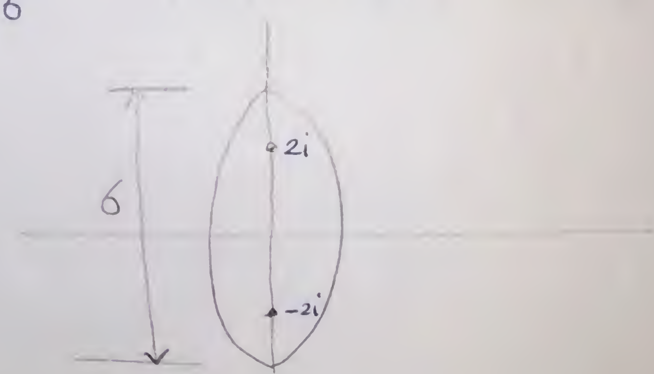
* Describe & graph

ⓐ $|z-i| = 2$

← دائره نصف قطرها 2 ومركزها $(0,1)$



ⓑ $|z+2i| + |z-2i| = 6$



ⓒ $\left| \dfrac{z-i}{z+i} \right| = 1$ ← ليست من الأشكال السابقة

$$\frac{|z-i|}{|z+i|} = 1$$

$$|x + iy - i| = |x + iy + i|$$

$$|x + i(y-1)| = |x + i(y+1)|$$

$$\sqrt{x^2 + (y-1)^2} = \sqrt{x^2 + (y+1)^2}$$ بتربيع الطرفين

$$\cancel{x^2} + (y-1)^2 = \cancel{x^2} + (y+1)^2$$

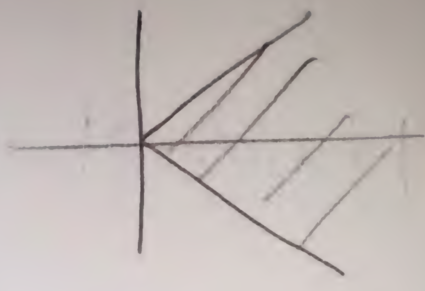
$$y^2 - 2y + 1 = y^2 + 2y + 1$$

$$-y = y \longrightarrow 2y = 0 \Longrightarrow \boxed{y = 0}$$

محور السينات

(d) $|\arg(z)| \leqslant \frac{\pi}{4}$

$\frac{-\pi}{4} \leqslant \arg(z) \leqslant \frac{\pi}{4}$



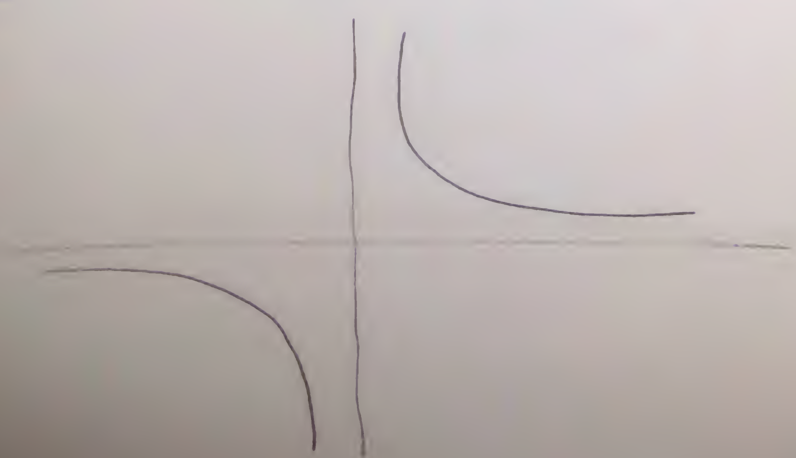
(e) $\mathrm{Im}\,(z^2) = 4$

$\mathrm{Im}\left[(x+iy)^2\right] = 4$

$\mathrm{Im}\left(x^2 + i2xy - y^2\right) = 4$

$2xy = 4 \Rightarrow xy = 2$

$y = \frac{2}{x}$



* Write the following functions on the form $w = u + iv$

$f(z) = u + iv$

(Z = x + iy) نستخدم
ماعدا $Ln(z)$ ، $Z^{n>2}$

$Z = re^{i\theta}$

$\boxed{1}$ $f(z) = Z^2$

$$= (x+iy)^2 = x^2 + i2xy - y^2$$

$$= \underbrace{(x^2 - y^2)}_{u} + i\,\underbrace{(2xy)}_{v}$$

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(2) $f(z) = \frac{1}{z} + z$

$$= \frac{1 + (x+iy)(x+iy)}{x+iy} = \frac{1 + x^2 + i2xy - y^2}{x+iy}$$

$x - iy$ بالضرب ×

$$= \underbrace{\frac{x + x^3 + xy^2}{x^2 + y^2}}_{u} + i\,\underbrace{\frac{-y + y^3 + x^2y}{x^2 + y^2}}_{v}$$

③ $f(z) = z \cdot e^{2z}$

$= (x+iy) \cdot e^{2(x+iy)}$

$= (x+iy)\, e^{2x} \cdot e^{2iy}$

$= e^{2x}\,(x+iy)\,\big(\cos(2y) + i\sin(2y)\big)$

$= xe^{2x}\cos(2y) + i\,xe^{x}\sin(2y) + iye^{2x}\cos(2y) - ye^{2x}\sin(2y)$

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④ $f(z) = Ln(z)$

$= Ln\,(re^{i\theta})$

$= Ln(r) + Ln\, e^{i\theta}$

$= \underbrace{Ln\, r}_{u} + \underbrace{i\theta}_{v}$

* Differentation

ـ النهاية تعادل المشتقة الأولى للدالة

$$\lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z} = f'(z)$$

$$\lim_{y} \lim_{x} = \lim_{x} \lim_{y}$$

example

* Show that $f(z) = 2x + ixy$ isn't diff.

$$\lim_{\Delta z \to 0} \frac{2(x+\Delta x) - i(x+\Delta x)(y+\Delta y) - 2x + ixy}{\Delta x + i\Delta y}$$

$$\lim_{\Delta z \to 0} \frac{2x + 2\Delta x - ixy - ix\Delta y - i\Delta x y - i\Delta x \Delta y - 2x + ixy}{\Delta x + i\Delta y}$$

$$\lim \frac{2\Delta x - ix\Delta y - i\Delta x y - i\Delta x \Delta y}{\Delta x + i\Delta y}$$

$$\lim_{\Delta y \to 0}\left[\lim_{\Delta x \to 0} \frac{2\Delta x - ix\Delta y - i\Delta xy - i\Delta x\Delta y}{\Delta x + i\Delta y}\right]$$

$$\lim_{\Delta y \to 0} \frac{-ix\Delta y}{i\Delta y} = \boxed{-x}$$

$$\lim_{\Delta x \to 0}\left[\lim_{\Delta y \to 0} \frac{2\Delta x - ix\Delta y - i\Delta xy - i\Delta x\Delta y}{\Delta x + i\Delta y}\right]$$

$$\lim_{\Delta x \to 0}\left[\frac{2\Delta x + i\Delta xy}{\Delta x}\right] = 2 - iy$$

$$\lim \neq \lim$$

↳ this function is not diff. #